The 100,000-sided polygon

Recommended Age Group: 8th-12th

Time commitment: 45 minutes

Supplies Needed for each group (4 people per group is recommended):

- Pencil and paper (preferably graph paper)
- Compass and protractor
- Calculator

Preparation:

- Depending on how familiar you are with the mathematics of this experiment, you may want to brush up on it (see **Background**). It is all elementary geometry, but I suppose some things are easy to forget.

Background:

This is one of my favorite activities because it teaches two things: First, it shows a tangible, arithmetic application of geometry, the absence of which is often the bane of some geometry students' existence. Second, it exposes them to the concept of limits and an improving approximation as x approaches infinity without really exposing them to any calculus. Let me explain:

The theory is very simple: If we consider a circle of radius 1, then we would expect the circumference to be pi*2*1, or simply 2*pi. Would the case be the same with, say, a 100,000-sided regular polygon? Certainly once a polygon gets to have around 100,000 sides, it certainly starts *looking* like a circle...

Remember that the each interior angle of a regular polygon is equal to 180 x (the number of sides (n) – 2)/n. For example, a regular pentagon has five interior angles all of measure 180*(5-2)/5 or 108 degrees. Now, the line from the vertex to the center (we will call this the radius) will bisect this interior angle. If we take the apothem (the distance of the line perpendicularly bisecting one of the sides and reaching the center of the polygon), the radius, and the half of the side that connects the two, you should have a right triangle whose angle at the base of the polygon is 180*(n-2)/2n.

With this triangle, we can find the length of a half-side of the polygon. Take the cosine of this angle and multiply it by the radius (the hypotenuse of the triangle), which is 1 in our case, and you will have the distance of half of one of the sides of the polygon. If we multiply this by 2n, then we should have the perimeter:

Perimeter = 2n*r*cos(180*(n-2)/2n)

Now, let's take a regular polygon of "radius" one (where the distance from the vertex to the center is one) and take the equation Perimeter/2*radius. For r = 1, this would be:

n*cos(180*(n-2)/2n)

Notice that as you make n bigger, this equation approaches pi! (For 100,000, one actually gets a decent approximation for pi.) The reason is the same reason that one can get a good approximation of a curved line by taking the values of very thin rectangles underneath the curve and adding them up. Theorhetically, as n->∞, p/2r->π!

Presentation:

For presentation, consider starting with a simple polygon like a square and ask if a circle can be squared. You may find it fun to relate that there was once a disease attributed to trying to square the circle called *morbus cyclometricus.*

Well, by the end of class today, everyone is going to catch *morbus cyclometricus*! Propose that the distance from the center of the square to one of its vertices be called its radius. Ask the students if they can find the perimeter of the sqaure based on the value of this radius. If they are unfamiliar, walk them through the process. Now have them divide the perimeter by 2 times the radius and they should get something that vaguely resembles pi. We have just "squared" a circle!

Have your students use the graph paper and protractors for drawing and approximating pentagons, hexagons, etc. See who can, by hand (except for evaluating cosine), get the best approximation for pi with their polygon drawings.

Finally, show them (or, if a more advanced class, have them do it) what happens with the 100,000-sided polygon. Explain that pi is not just about circles, but it is found *everywhere*, which is what makes it so unique.